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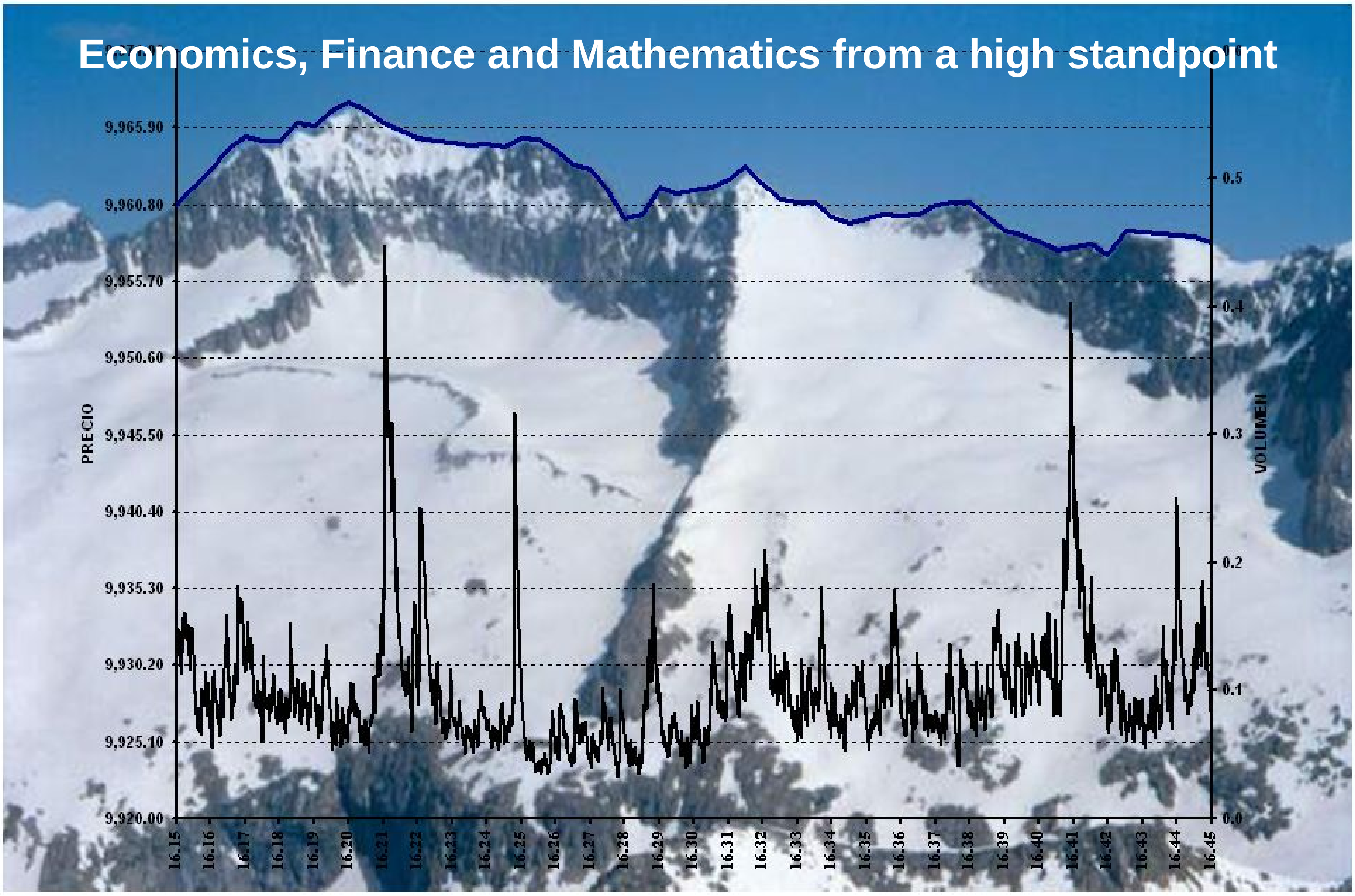

Interest Rate Risk Management Measures for Bonds with Embedded Options from Different Interest Rate Models

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INTEREST RATE RISK MANAGEMENT MEASURES FOR BONDS WITH EMBEDDED OPTIONS FROM DIFFERENT INTEREST RATES MODELS

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Abstract: This paper examines the behaviour of the interest rate risk management measures for bonds with embedded options, studying the factors it depends on. The two most common embedded options are call provisions and put provisions. Traditional sensitivity measures of interest rare risk are not suitable for these bonds as they do not consider the possibility of option exercise. In this case, Effective Duration (ED) and Effective Convexity (EC) are used. The uncertainty of future interest rates implies that a model of interest rates is required to price. We use the well-known Ho and Lee (1986) and Black, Derman and Toy (1990) models and price a callable and a putable bond during their lifetime. The estimates of ED and EC are more stable the wide is the interest rate change considered. Their values depend on forward rates and volatility and are highest for HL than for BDT. The differences between models fall down in high volatility scenarios.

Keywords: bonds with embedded options, non-arbitrage interest rates models, effective duration, effective convexity
JEL Classification: E43; F31; G12; G13; G15

1 Introduction

The aim of this paper is to analyse the most popular interest rate risk management measures for bonds with embedded options and study the factors they depends on. We price two Spanish corporate bonds during all their lifetime applying two consistent term structure of interest rates models (Ho and Lee, 1986, and Black, Derman and Toy, 1990). We estimate different interest risk management measures and examine the main factors which determine their behaviour, such as volatility of interest rates, shape of the yield curve and yield curve changes.

The two most common types of embedded options are call provisions and put provisions. Callable bonds may be redeemed by the issuer before the scheduled maturity date. The call feature benefits the borrower since it permits to replace the bond issue with a lower-interest-cost issue when the market rates fall. Thus, callable bonds carry higher yields than bonds that cannot be retired before maturity. In contrast, a putable bond grants the bondholder the right to sell the issue back to the issuer at par value on designed dates.

Traditional sensitivity measures of interest rate risk are not suitable for option-embedded bonds because they do not consider the possibility of option exercise. The performance of callable or putable bonds is analysed looking at their risk originated by changes in the underlying variables, such as volatility or yield curve changes. Effective Duration or Option Adjusted Duration (ED), and Effective Convexity or Option Adjusted Convexity (EC), are used. On the other hand, the yield-to-maturity is replaced by the yield-to-worst and the Option Adjusted Spread (OAS) appears as a new measure.

In our paper we make two contributions to the literature. Firstly, we analyse the factors which determine the bond pricing and the effectiveness of the sensitivity measures using actual and extensive sample data. Secondly, we apply the OAS methodology, widely employed by practitioners in developed markets, to price and analyse bonds with embedded options in the Spanish case.

2 Consistent Term Structure of Interest Rates Models

To price the assets we apply two consistent term structure of interest rates models, Ho and Lee (1986) and Black, Derman and Toy (1990). They are the most popular models among investors and academics in the financial industry to price interest rate derivatives.

2.1 The Ho-Lee Model (1986)

Ho and Lee (1986) is the first consistent term structure of interest rates model and is presented as an alternative to equilibrium models (Vasicek, 1977; Cox, Ingersoll and Ross, 1985). It proposes a general methodology to price a wide range of interest rates contingent claims. The inputs of the model are the yield curve and the short rate volatility. The main limitation is that interest rates are normally distributed so we can get negative values.

The short-interest rate dynamic can be represented by the expression

$$dr = \theta(t)dt + \sigma dz \quad (1)$$

where $\theta\,(t)$ is the drift of the process, that depends on the time t and the slope of the forward curve in 0, ($f(0, t)$) and the process volatility (σ):

$$\theta(t) = \frac{\partial f(0,t)}{\partial t} + \sigma^2 t \quad (2)$$

On the other hand, the short rate volatility is a constant for all the terms

$$\sigma_R\,(t, s) = \sigma \quad (3)$$

so to calibrate the model we introduce the one-month rate volatility as an input[1].

2.2 The Black-Derman-Toy Model (1990)

The Black, Derman and Toy (1990) model assumes that interest rates follow a lognormal distribution. To implement the model we follow the forward process developed by Jamshidian (1991) that proves that the level of the short rate in t can be estimated from the expression

$$r(t) = U(t)\exp(\sigma(t)z(t)) \quad (4)$$

where $U(t)$ is the median of the short rate distribution in t, σ(t) is the short rate volatility and $z(t)$ represents is the brownian movement.

In this case we consider a term structure of volatilities, so the volatility is different for each term and they are related as we can see in the following equation

$$\sigma_R(i)\sqrt{\Delta t} = \frac{1}{2}\ln\frac{\ln P_U(i)}{\ln P_D(i)} \quad (5)$$

P_D is the bond price is interest rates fall down and P_U is the bond price when the interest rates increase. An equivalent formulation is

$$P_D(i) = P_U(i)^{\exp(-2\sigma_R(i)\sqrt{\Delta t})} \quad (6)$$

The models implementation is made from the binomial method. We built binomial trees for the short rate with monthly time steps consistent to the estimated Spanish yield curve following the procedure proposed by Skinner (2005). Estimates are made from the daily trading of Spanish Treasury debt securities using the Nelson and Siegel (1987) model weighted by duration.

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[1] Clelow y Strickland (1998), p. 209.

3 Bonds with embedded options and interest rate risk management measures: ED and EC.

3.1 Bonds with embedded options

We can distinguise two main types of bonds with embedded options: callable bonds and putable bonds. A callable bond is a bond that can be redeemed by the issuer before its maturity date and a putable bond can be sold by the bond holder before its maturity date. Hence, buying a callable bond comes down to buying an option-free bond and selling a call option to the issuer of the bond, so the value of a callable bond can be estimated from the expression

$$P_t\ callable = P_t - Premium\ call_t \quad (7)$$

Similarly, buying a putable bond comes down to buying an option-free bond as well as a put option, so its value can be calculated

$$P_t\ putable = P_t + Premium\ put_t \quad (8)$$

The embedded option can be exercised from a specific date on (American option) or on a specific date (European option), depending on the bond, at a specific price (strike price).

3.2 Effective Duration (ED), Effective Convexity (EC) and Option Adjusted Spread (OAS).

The cash-flow structure of a bond with an embeddded option is directly impacted by the level of interest rates so the traditional modified duration and convexity measures are not relevant for such a bond. Instead, effective duration (ED) and effective convexity (EC) are used. The formulas are given by

$$DE = \frac{P_D - P_U}{2P_0(\Delta y)} \quad (9) \qquad CE = \frac{P_D + P_U - 2P_0}{2P_0(\Delta y)^2} \quad (10)$$

To estimate these interest risk management measures we follow the usual procedure[2], shifting the Treasury yield curve introduced as an input +/-25 b.p. and +/-100 b.p. to recalibrate the models in the traded dates. In addition, we have to calculate the Option Adjusted Spread (OAS) for each date. The OAS is the constant spread that, when added to all short-term interest rates on the binomial tree equalizes the theoretical price of a bond to its market price.

[2] The procedure is described in Fabozzi (2002) and Martellini, Priaulet, Priaulet (2003).

4 Sample description and estimation procedure

We have chosen the two most actively traded corporate bond issues with embedded provisions in the Spanish corporate fixed income market AIAF along the period 1993-2004. One of them contains a call option and the other bond a put option. Both provisions are European options.

Issuer	Banco de Crédito Local	Túnel del Cadí
Issuance Date	14/12/1993	31/05/1994
Maturity Date	01/07/2003	31/05/2004
Annual Coupon Rate (%)	8.4	9.85
Amount Outstanding	€ 178 million	€ 48 million
Rating	Aa3	No Rated
% Traded Days	9.5%	5.1%
Daily Trading Volume	€ 6.5 million	€ 1.0 million
Option Type	Call (European)	Put (European)
Option Strike Price (%)	100	100
Option Strike Date	01/07/1998	31/05/2000
No. Observ. until Option	106	101
No. Observations	106	128

Table 1. *Main features of the issues.*

Hence, we calibrate the HL and the BDT Models to the Spanish Treasury yield curve. Estimates are made from the daily trading of Spanish Treasury debt securities using the Nelson and Siegel (1987) model weighted by duration.

To calculate sensitivity measures for bonds with embedded options we apply the Fabozzi (2002) procedure, similar to Martellini, Priaulet, Priaulet (2003). First, we calculate the theoretical price of the bonds from the two yield curve models. Second, we obtain the OAS for all days which issues are traded along the period 1993-2004. Third, we shift the on-the-run yield curve up and down by a small number of basis points and construct new binomial interest rate trees. Fourth, we add the constant OAS to each knot of the new interest rates trees. Fifth, we use the adjusted trees to determine the value of bonds from which we calculate ED and EC (eq. 9 and 10).

Also, we compute for all the sample period the modified duration and convexity assuming two possibilities. In the first case, the option is not exercised so we have the Modified Duration and Convexity to Maturity. In the second one, we suppose that the option is exercised so we have the Modified Duration and Convexity to Call/Put. We compare these new measures with ED and EC.

5 Results

In figure 1 we represent the results for both consistent models. In the top of the graph we have the estimates for the Effective Duration calculated from a shifting of +/-100 b.p. in the Treasury yield curve. The Option Attractiveness for the investor, a measure we have defined, is in the bottom of the graph and it is calculated from the expression

Option Attractiveness = Forward (t,T) + OAS – Coupon Rate (11)

where the Forward (t,T) is the prediction, assuming the expectative theory is true, of future interest rates for the period between the strike date t and the maturity date T (t+5 for BCL and t+4 for TC), OAS is the media of the session and we subtract the Coupon Rate. Hence, for callable bonds, the option is attractive when the O.A. gets negative values while for putable bonds the option is attractive when the O.A. is positive.

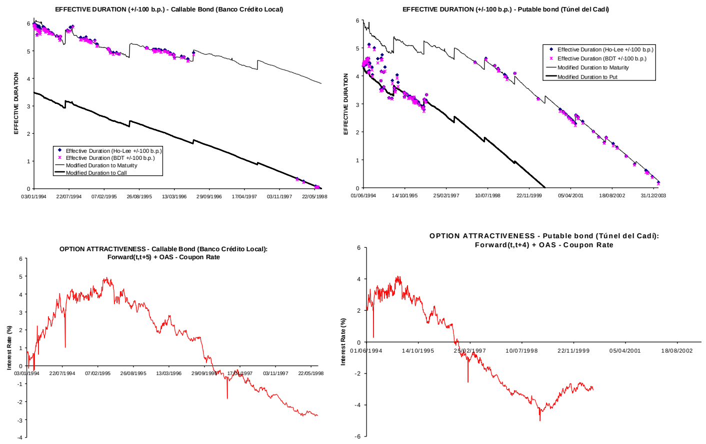
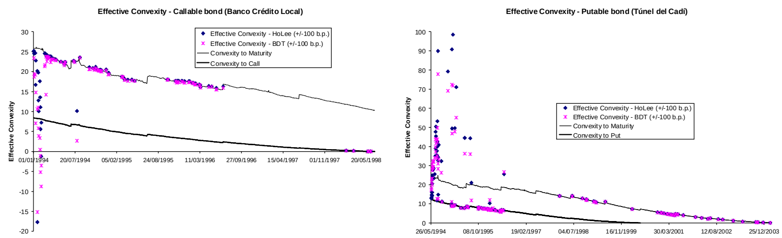


Figure 1. *Effective Duration for BCL and TC from a shifting of +/-100 b.p.compared with the Option Attractiveness (Forward (t,T)+OAS-Coupon Rate)*

We have represented the Effective Convexity in Figure 2 and we can see a similar pattern for the results. The first days we have unstable data but the probability of option exercise approach the results to the Convexity to Call (BCL issue) and to the Convexity to Maturity (TC issue).



The differences between models occur too for the premium options. In general, we can see that HL model generate bigger values than the BDT model for ED and EC, and smaller ones for the option values. The deviations are resumed in table 2.

	Effective +/-25 bp	Duration +/-100bp	Effective +/-25 bp	Convexity +/-100bp	Option Price (*)
Callable bond					
- Average	0.0388	0.0490	1.4863	1.6681	-0.1288
- Median	0.0373	0.0433	0.2307	0.2423	-0.1227
- St.Dev.	0.0258	0.0362	3.8977	3.3989	0.1274
Putable bond					
- Average	0.0062	0.0268	-0.8147	0.6357	-0.0649
- Median	0.0215	0.0284	0.0906	0.1368	-0.0535
- St.Dev.	0.0736	0.0511	20.0785	5.0451	0.0463

Table 2. *Differences in ED, EC and Option Prices, calculated as the value of the variable from HL minus the value from BDT* .

Below we have defined regressions models to analyse the differences, and we resume the results in the conclusions.

6 Conclusions

The main objective of our paper is to a analyze and to explain the factors that determine the interest risk management measures commonly used by investors in bonds with embedded options, Effective Duration and Effective Convexity, both estimated from two consistent term structure of interest rates models (HL and BDT), shifting the TSIR used as an input +/-25 b.p. and +/-100 b.p.. We have chosen the two most actively traded corporate bond issues with embedded provisions in the Spanish corporate fixed income market AIAF in the period 1993-2004.

We can see that the differences between ED and EC and the traditional ones, are produced by the probability of option exercise in every moment. Then, when the option is in-the-money, the ED and EC values are smaller than the duration and convexity to maturity and they approach to duration and convexity to the strike date.

The option price mainly depends on the interest rates volatility and the future rates. So we can see that, when the forward (t, T) increases, the call premium decreases and grows up the put premium. This happens because when the interest rates are bigger than the coupon rate, the put is in the money (the bondholder will sell the bond and will acquire another one with bigger return), while the issuer of a bond with a call option will not refinance if the interest rates are above the coupon rate. The interest rate volatility is directly related with the premium of both types of options.

When compare the interest rate models, BDT and HL, we can see that the HL model generate bigger values than the BDT model for ED and EC, and smaller ones for the option values. Thus, for Ho Lee we have that the estimates of DE are the nearest of Modified Duration (MD) to maturity, because the option values from HL are the smallest. On the other hand, the differences between models are slightly smaller when ED and EC are estimated from shifts of the TSIR in an amount of +/-25 b.p. Nevertheless, the results are much more stable and consistent when the risk measures are estimated from shifts of the TSIR of +/-100 b.p.. We can see that the interest rate volatility is the key factor in determine these differences. So, the bigger the volatility of interest rates, the nearer are the results that we obtain with both models. This happens because the volatility we introduce as an input in the Ho Lee Model is the short rate volatility so it considers that σ is a constant for all the terms while the BDT Model includes the term structure of volatilities as an input. Thus, the choice of the interest rate model to estimate the ED and the EC and to calculate the option prices in bonds with embedded provisions is specially relevant in stable scenarios of interest rates.

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